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Linear-time admission control for elastic scheduling

Marion Sudvarg[1] · Chris Gill[1] · Sanjoy Baruah[1]

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Abstract
Prior algorithms that have been proposed for the uniprocessor implementation of systems of elastic tasks have computational complexity quadratic ($O(n^2)$) in the number of tasks n, for both initialization and for admitting new tasks during run-time. We present a more efficient implementation in which initialization takes quasi-linear ($O(n \log n)$), and on-line admission control, linear ($O(n)$), time.

Keywords Preemptive uniprocessor scheduling · Elastic tasks · Admission control

1 Introduction

The elastic recurrent real-time workload model (Buttazzo et al. 1998, 2002) provides a framework for dealing with overload by compressing (i.e., reducing) the effective utilizations of individual tasks until the cumulative utilization falls below the utilization bound that can be accommodated. Each task $\tau_i = (U_i^{\min}, U_i^{\max}, E_i)$ is characterized by the minimum amount of utilization $U_i^{\min}$ that it must be provided and the maximum amount $U_i^{\max}$ that it is able to use, as well as an additional elasticity parameter E_i that "*specifies the flexibility of the task to vary its utilization*" (Buttazzo et al. 1998). Given a system $\Gamma = \{\tau_1, \tau_2, \ldots, \tau_n\}$ of n such elastic tasks, the objective is to assign each task τ_i a utilization U_i, $U_i^{\min} \le U_i \le U_i^{\max}$, such that

✉ Sanjoy Baruah
baruah@wustl.edu

Marion Sudvarg
msudvarg@wustl.edu

Chris Gill
cdgill@wustl.edu

1 Washington University in Saint Louis, Campus Box 1045, Saint Louis, MO 63130, USA

(1) $\sum_{i=1}^{n} U_i$ is as large as possible but bounded from above by a specified constant U_d which denotes the maximum cumulative utilization that can be accommodated; and (2) if $U_i > U_i^{\min}$ and $U_j > U_j^{\min}$ then U_i and U_j must satisfy the relationship[1]

$$\left(\frac{U_i^{\max} - U_i}{E_i}\right) = \left(\frac{U_j^{\max} - U_j}{E_j}\right) \tag{1}$$

A task system Γ for which such U_i exist for all the tasks is said to be feasible. An algorithm was presented in Buttazzo et al. (1998) Fig. 3 for determining feasibility and of computing the appropriate values for the utilizations —the U_i's— of feasible systems in $O(n^2)$ time. Essentially this same algorithm was also repurposed in Buttazzo et al. (1998) for admission control: for determining whether a new task seeking to join an already-executing system could be admitted without compromising feasibility, and if so, recomputing the utilization values for the new task as well as for all preëxisting ones. Extensions to elastic scheduling that were proposed by Chantem et al. (2006, 2009) reformulate the problem of determining the utilizations as a quadratic programming problem. This allows the iterative technique in Buttazzo et al. (1998) to be applied to a more general class of problems. However, this reformulation continues to have quadratic time-complexity. In this short note we present a more efficient implementation of the algorithm of Buttazzo et al. (1998) Fig. 3 that determines feasibility and computes the U_i values in $O(n \log n)$ time, and does admission control in $O(n)$ time.

2 Overview of Prior Results

Let Γ denote a feasible task system with $E_i > 0$ for all tasks[2] $\tau_i \in \Gamma$, and consider the U_i values that bear witness to this feasibility (i.e., each U_i either equals $U_i^{\min}$, or satisfies Expression 1). The tasks in Γ may be partitioned into two classes Γ_{VARIABLE} (those tasks for which $U_i > U_i^{\min}$, and which can therefore have their utilizations "varied" –compressed– further if necessary) and Γ_{FIXED} (those for which $U_i = U_i^{\min}$; i.e., their utilizations are now "fixed"). It has been shown (Buttazzo et al. 1998, Eqn. 8) that for each $\tau_i \in \Gamma_{\text{VARIABLE}}$

$$U_i = U_i^{\max} - \left(\frac{U_{\text{SUM}} - \left(U_d - \Delta\right)}{E_{\text{SUM}}}\right) \times E_i \tag{2}$$

where $U_{\text{SUM}} = \left(\sum_{\tau_i \in \Gamma_{\text{VARIABLE}}} U_i^{\max}\right)$ and $E_{\text{SUM}} = \left(\sum_{\tau_i \in \Gamma_{\text{VARIABLE}}} E_i\right)$ respectively denote the sum of the $U_i^{\max}$ parameters and the E_i parameters of all the tasks in Γ_{VARIABLE},

[1] For tasks τ_i having $E_i = 0, U_i = U_i^{\min}$, and therefore the relationship needs not be satisfied.

[2] All tasks τ_i with $E_i = 0$ must have $U_i \leftarrow U_i^{\max}$ in order to satisfy Expression 1; we assume this is done in a pre-processing step, and the value of U_d updated to reflect the remaining available utilization.

and $\Delta = \left(\sum_{\tau_i \in \Gamma_{\text{FIXED}}} U_i^{\min}\right)$ denotes the sum of the $U_i^{\min}$ parameters of all the tasks in Γ_{FIXED}.[3] Given a set of elastic tasks Γ, the algorithm of Buttazzo et al. (1998) Fig. 3 starts out computing U_i values for the tasks assuming that they are all in Γ_{VARIABLE} — i.e., their U_i values are computed according to Expression 2. If any U_i so computed is observed to be smaller than the corresponding $U_i^{\min}$ then that task is moved from Γ_{VARIABLE} to Γ_{FIXED}, the values of U_{SUM}, E_{SUM}, and Δ are updated to reflect this transfer, and U_i values recomputed for all the tasks. The process terminates if no computed U_i value is observed to be smaller than the corresponding $U_i^{\min}$. It is easily seen that one such iteration (i.e., computing U_i values for all the tasks) takes $O(n)$ time. Since an iteration is followed by another only if some task is moved from Γ_{VARIABLE} to Γ_{FIXED} and there are n tasks, the number of iterations is bounded from above by n. The overall running time for the algorithm of (Fig. 3, Buttazzo et al. (1998)) is therefore $O(n^2)$.

Algorithm 1: Elastic_Compression(Γ, U_d)

Input: A list Γ of elastic tasks sorted in non-decreasing order of their ϕ_i parameters (see Expression 3) and a desired utilization U_d
Output: Feasibility and the list Γ with computed U_i values

$U_{\text{SUM}} = 0$; $E_{\text{SUM}} = 0$; $\Delta = 0$
forall $\tau_i \in \Gamma$ **do**
    $U_{\text{SUM}} = U_{\text{SUM}} + U_i^{\max}$
    $E_{\text{SUM}} = E_{\text{SUM}} + E_i$
end
forall $\tau_i \in \Gamma$ **do**
    if $\left(U_i^{\max} - \frac{U_{\text{SUM}} - (U_d - \Delta)}{E_{\text{SUM}}} \times E_i \le U_i^{\min}\right)$ **then**
        //Task τ_i is no longer compressible – it's in Γ_{FIXED}
        $U_i = U_i^{\min}$ //Since $\tau_i \in \Gamma_{\text{FIXED}}$
        $\Delta = \Delta + U_i^{\min}$ //This additional amount of utilization is allocated to tasks in Γ_{FIXED}
        if $(\Delta > U_d)$ **then return** INFEASIBLE;
        //Cannot accommodate the minimum requirements
        $U_{\text{SUM}} = U_{\text{SUM}} - U_i^{\max}$ //Since τ_i is removed from Γ_{VARIABLE}
        $E_{\text{SUM}} = E_{\text{SUM}} - E_i$ //As above — since τ_i is removed from Γ_{VARIABLE}
        $i = i + 1$ //Proceed to considering the next task...
    else
        //Remaining tasks are all compressible (i.e., in Γ_{VARIABLE})
        $U_i = U_i^{\max} - \frac{U_{\text{SUM}} - (U_d - \Delta)}{E_{\text{SUM}}} \times E_i$ // As per Expression 2
    end
end
return FEASIBLE

[3] Observe that Δ equals the amount of utilization that is allocated to the tasks in Γ_{FIXED}; therefore $(U_d - \Delta)$ represents the amount available for the tasks in Γ_{VARIABLE}, and $\left(U_{\text{SUM}} - (U_d - \Delta)\right)$ the amount by which the cumulative utilizations of these tasks must be reduced from their desired maximums. As shown in the RHS of Expression 2, under elastic scheduling this reduction is shared amongst the tasks in proportion to their elasticity parameters: τ_i's share is (E_i/E_{SUM}).

3 Our Approach

Let us define an attribute ϕ_i for elastic task τ_i as follows:

$$\phi_i \stackrel{\text{def}}{=} \left(\frac{U_i^{\max} - U_i^{\min}}{E_i} \right) \tag{3}$$

We will prove a result (Theorem 1 below) that allows us to conclude that in the algorithm of (Fig. 3, Buttazzo et al. (1998)), ***tasks may be "moved" from Γ_{VARIABLE} to Γ_{FIXED} in order of their ϕ_i parameters***.

Assuming that the tasks are indexed in a linked list such that $\phi_i \le \phi_{i+1}$ for all $i, 1 \le i < n$, we can then simply make a *single* pass through all the tasks from τ_1 to τ_n, identifying, and computing U_i values for, all the ones in Γ_{FIXED} before any of the ones in Γ_{VARIABLE}. With appropriate book-keeping (see the pseudo-code in Algorithm 1) this can all be done in a single pass in $O(n)$ time. The cost of sorting the tasks in order to arrange them according to non-increasing ϕ_i parameters is $O(n \log n)$, and hence dominates the overall run-time complexity: determining feasibility and computing the U_i parameters can be done in $O(n \log n) + O(n) = O(n \log n)$ time.

Admission control – determining whether it is safe to add a new task and recomputing all the U_i parameters if so – requires that the new task be inserted at the appropriate location in the already sorted list of preëxisting tasks — this can be achieved in $O(n)$ time. Once this is done, the U_i values can be recomputed in $O(n)$ time by the pseudo-code in Algorithm 1. Similarly, removing a task from the system and recomputing the U_i values also takes $O(n)$ time since sorting is not needed.

4 A Technical Result

We now present the main technical result in this short note.

Theorem 1 *If $\tau_i \in \Gamma_{\text{FIXED}}$ and $\phi_i \ge \phi_j$ then $\tau_j \in \Gamma_{\text{FIXED}}$.*

Proof Consider some iteration of the algorithm of (Fig. 3, Buttazzo et al. (1998)) such that τ_i and τ_j both start out in Γ_{VARIABLE}, but τ_i is determined to belong in Γ_{FIXED} in this iteration. This implies that $U_i^{\min}$ is at least as large as the value of U_i that is computed according to Expression 2:

$$U_i^{\min} \ge U_i^{\max} - \left(\frac{U_{\text{SUM}} - \left(U_d - \Delta\right)}{E_{\text{SUM}}} \right) \times E_i$$

By algebraic simplification of the above, we have

$$\left(\frac{U_{\text{SUM}} - \left(U_d - \Delta\right)}{E_{\text{SUM}}} \right) \ge \left(\frac{U_i^{\max} - U_i^{\min}}{E_i} \right) \tag{4}$$

Note that the LHS of Expression 4 does not contain any term specific to τ_i and so is the same for all the tasks in Γ_{VARIABLE} for this iteration, and that the RHS is simply ϕ_i. Since $\phi_i \geq \phi_j$ (as per the statement of the theorem), we may conclude by the transitivity of the $\geq$ operator on the real numbers that the LHS of Expression 4 would also be $\geq \phi_j$; equivalently, the value of $U_j^{\min}$ is no smaller than the value of U_j that is computed according to Expression 2, and as a consequence τ_j, too, should be moved to Γ_{FIXED}. □

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Marion Sudvarg is a PhD student studying computer science at Washington University in St. Louis. He earned his master's degree in computer science from Washington University in St. Louis, with an emphasis on data mining and machine learning. His current research interests are in developing robust, adaptable real-time computing systems. Additionally, he works with the ADAPT collaboration, developing real-time data analysis algorithms for multi-messenger astronomy.

Chris Gill research focuses on assuring properties of cyber-physical, real-time, and embedded systems in which software complexity, interactions with unpredictable environments, and heterogeneous platforms demand novel solutions that are grounded in sound theory. A major goal of his work is to assure that constraints on timing, memory footprint, fault-tolerance, and other system properties can be met across heterogeneous applications, operating environments, and deployment platforms. He has led or contributed to the development, evaluation, and open source release of numerous real-time systems research platforms and artifacts, including: the Kokyu real-time scheduling and dispatching framework that was used in several AFRL and DARPA projects and flight demonstrations; the nORB small-footprint real-time object request broker; the CyberMech platform (collaborative with Purdue University) for parallel Real-Time Hybrid Simulation; and the RT-Xen real- time virtualization research platform, from which the RTDS scheduler was transitioned into the Xen software distribution.

Sanjoy Baruah is the Hugo F. & Ina Champ Urbauer Professor of Computer Science & Engineering at Washington University in St. Louis. His research interests and activities are in real-time and safety-critical system design, scheduling theory, and resource allocation and sharing in distributed computing environments.